***Question one***: find the global minimum of in the interval [0, 10]

**IMAT 5232**

*Computational Intelligence Optimisation*

**Interim Assessment One**

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First-order derivative :

* or
* (power rule)

is never equal to 0 when (no stationary point(s) – constantly increasing)

is never zero given the implications of the denominators operation, meaning that there are no solutions for (no null derivatives); where has no stationary point(s) when . Relating to the interval provided, it can be assumed that the lower bound of the search space 0 is potentially the objective-functions global minima (critical point where ), when aligned with the functions first-order derivative, for which presents a positive, continuous gradient or slope relation; the function can be said to be constantly increasing.

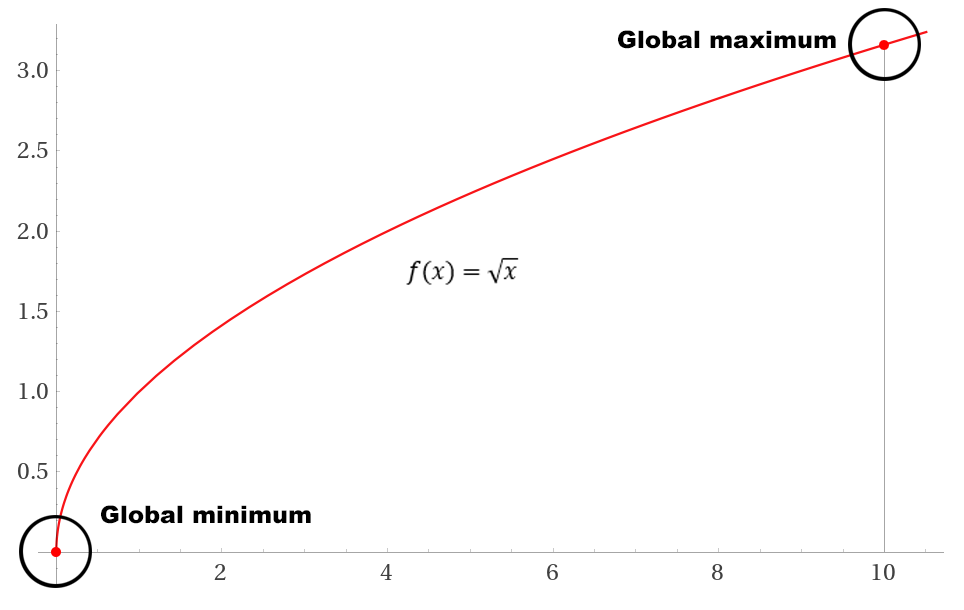
Second-order derivative :

* (exponent rule)
* (power rule)

Given that the first-order derivative of the objective-function does not satisfy a stationary point, calculating the second-order derivative for said function is not necessary but is rather purposed for demonstrating the procedure.

Search space bound check (critical point(s) endpoints of the domain), where and = 10: , which is inevitably greater than 0, for when .

Therefore: global minimum = 0 when and global maximum 3.1623 when , for the given objective-function when is subjected to the interval [0, 10]; alternatively, as when .



***Question two***: find the global minimum of in the interval [1, 10]

First-order derivative :

* (natural logarithm)

is never equal to 0 when (no stationary point(s) – constantly increasing)

is never zero given the implications of the divisive operation, meaning that there are no solutions for (no null derivatives); where has no stationary point(s) when . Corresponding with the interval provided, it can be assumed that the lower bound of the search space 1 is potentially the objective-functions global minima (critical point where ), when aligned with the functions first-order derivative, for which presents a positive, continuous gradient or slope relation; the function can be said to be constantly increasing, parallel with the preceding example.

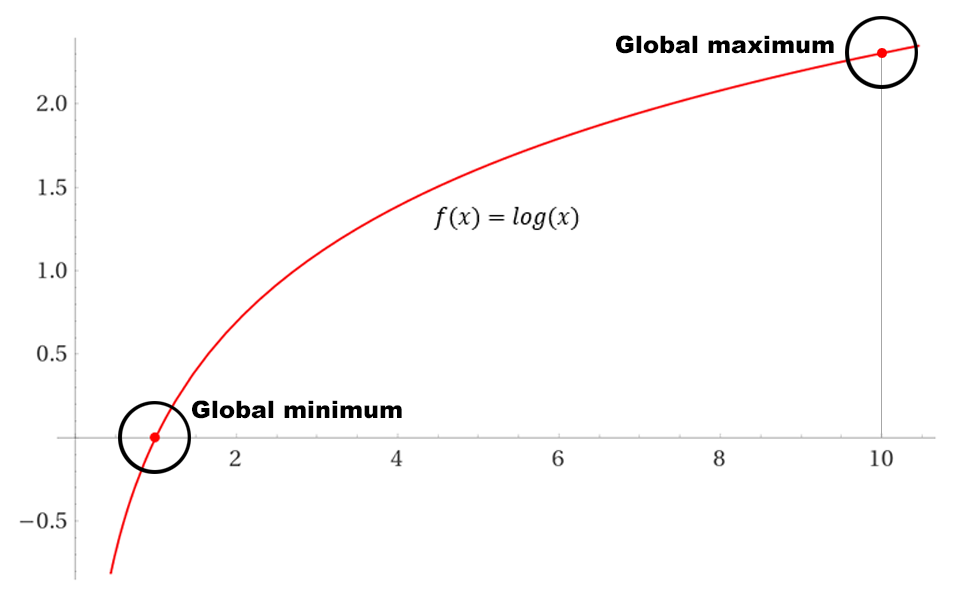
Second-order derivative :

* (exponent rule)
* (power rule)

Alike the prior example, as the first-order derivative of the objective-function does not satisfy a stationary point, calculating the second-order derivative for said function is not necessary but is yet again purposed for demonstrating the procedure, if proving the extremum of a stationary point were possible.

Search space bound check (critical point(s) endpoints of the domain), where and = 10: , which is inevitably greater than 0, for when .

Therefore: global minimum = 0 when and global maximum 2.3026 when , for the given objective-function when is subjected to the interval [1, 10]; alternatively, as when .



***Question three***: find the global minimum of in in the interval [−1, 1]

First-order derivative :

* (power rule)

0 when (one stationary point)

Second-order derivative :

* (power rule)

0 when (one stationary point)

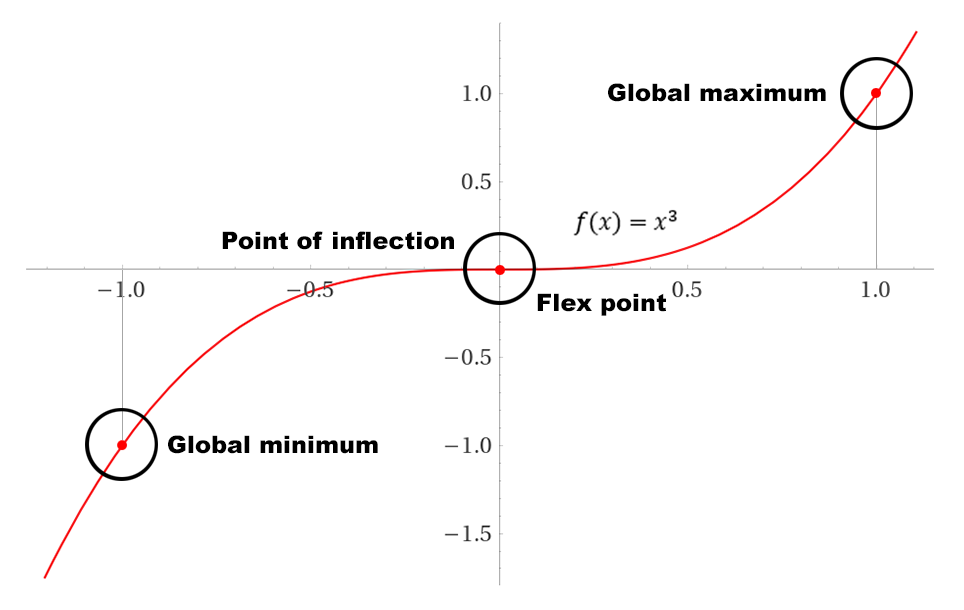
Third-order derivative :

0 when (stationary point is a point of inflection)

is a flex point (point of inflection or transition in curve concavity), given that the derivative of the function when is neutrally 0, in focus of the interval [-1, 1], when the derivative of the function is positive and when the derivative of the function is negative. In realisation of said relation, when , it is regarded as a stationary point but not an extremum as the function slope is continually increasing, from which there are no maximum or minimum extrema; this is proven by the third-order derivative: , which evidently does not have a null derivative as 0 denotes.

Search space bound check (critical point(s) and endpoints of the domain), where , when : , when : and when : .

Therefore: point of inflection = 0 when , global minimum = when and global maximum = 1 when , for the given objective-function when is subjected to the interval [-1, 1]; alternatively, as when .



***Question four***: find the global minimum of

Solve the gradient of the objective-function:

0

Solve the system of equations:

* (division by a multiple of three - simplify)

Thus: or

* (substitute into equation )

* (division by a multiple of three - simplify)

Hence:

* or

Therefore:

Conclusively: there exists two critical points for the objective-function in the interval , and

Solve the Hessian matrix:

Find the characteristic polynomial (computing the determinant) of the Hessian matrix:

Therefore: , where point (0, 0) is not an extremum, but a saddle point alternatively.

Find the characteristic polynomial (computing the determinant) of the Hessian matrix:

Therefore: and , where point () is an extremum, specifically a local minimum.

Given that the point is the only local minimum within the interval , it thereby represents the global minimum of the domain also, where its functional value when ; as calculates.